

# Extreme Climatic Events: Impacts for Ireland

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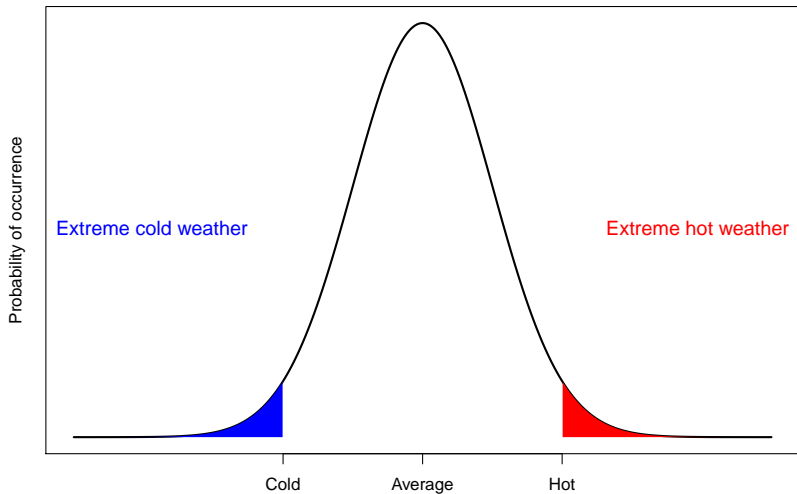
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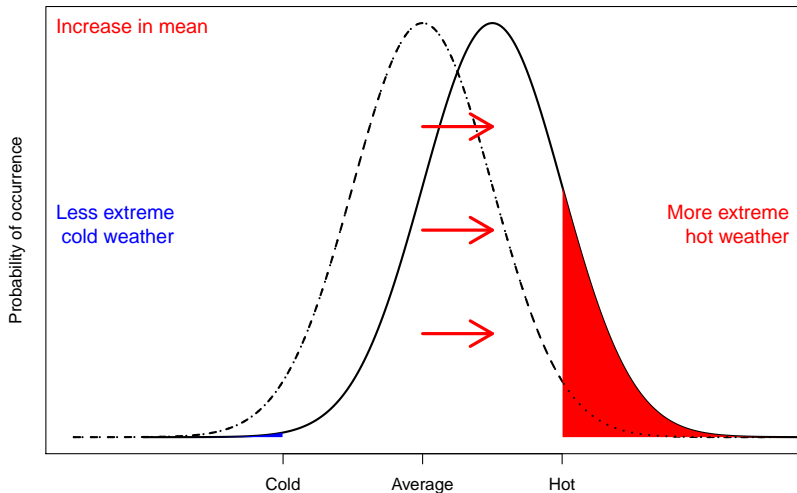
# What is this talk about?

- ▶ We want to predict extreme temperatures, wind speeds, and rainfall for Ireland up to 2100
- ▶ We have a large set of data covering 50 years of weather in Ireland
- ▶ We have some down-scaled climate model runs of weather/climate up to 2100
- ▶ We want to use all this information together to estimate past, present and future extremes whilst accounting for uncertainty
- ▶ In this talk I'm only going to cover changes in past extremes

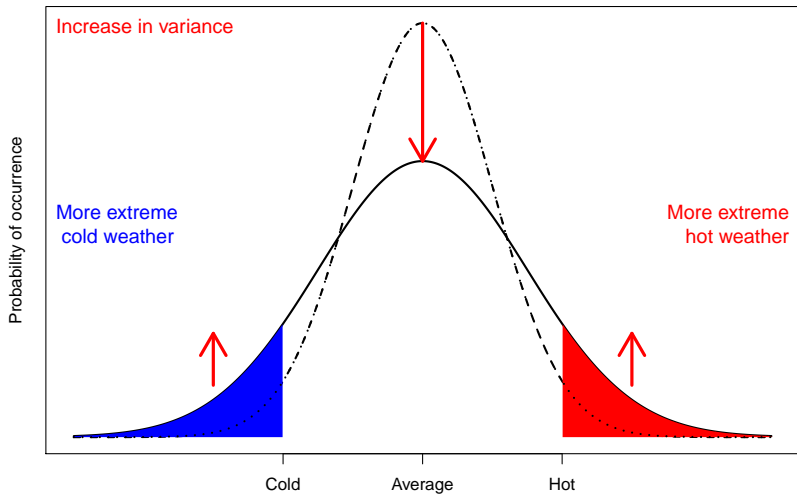
# What is climate and what are extremes?



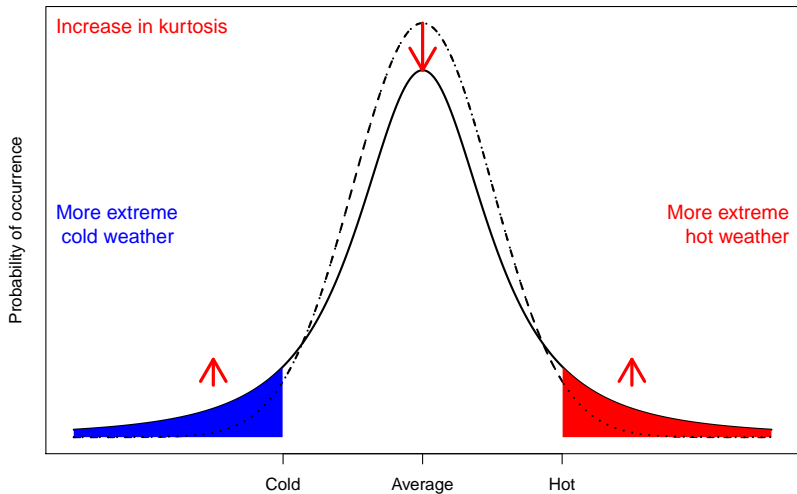
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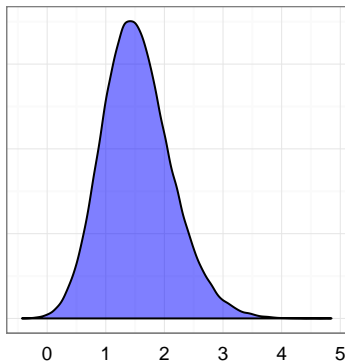
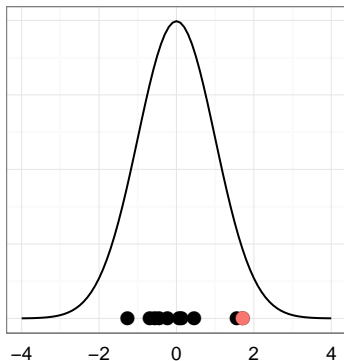


## Some comments

- ▶ These plots are still an **over-simplification** (e.g. skew/multi-modal/multivariate distributions)
- ▶ Thinking about climate changes only in terms of the mean is not sufficient in determining the frequency or size of extreme events
- ▶ If we are interested in extremes we need to focus on the **tail behaviour** of the probability distribution
- ▶ The discipline of **Extreme Value Theory** was invented to determine changes in maxima (or minima) of series. It is not widely taught

# The amazing Fisher-Tippett-Gnedenko theorem

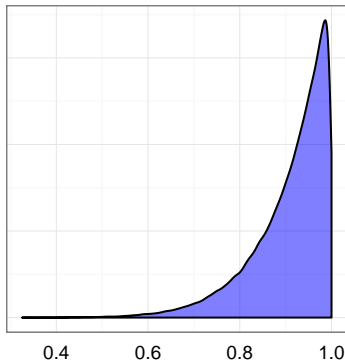
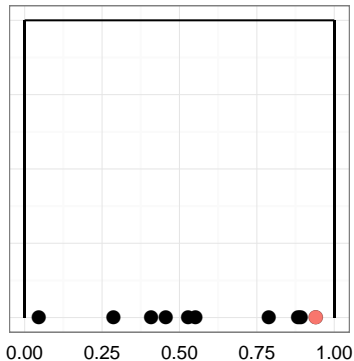
- ▶ A remarkable theorem: repeatedly take the maxima of any distribution and you will end up with a **Generalised Extreme Value** (GEV) distribution





# The amazing Fisher-Tippett-Gnedenko theorem

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# Extreme Value Theory

- ▶ We can use the GEV distribution to model the **tail behaviour** of the distribution
- ▶ There are two key parameters, the **scale**, which measures how spread out the distribution is, and the **shape**, which measures how long-tailed the distribution is
- ▶ If the shape is **negative**, then there is a maximum value in the data. If the shape is **positive**, then the data are long-tailed
- ▶ Positive shape parameters for climate are **dangerous** - more extreme extremes!

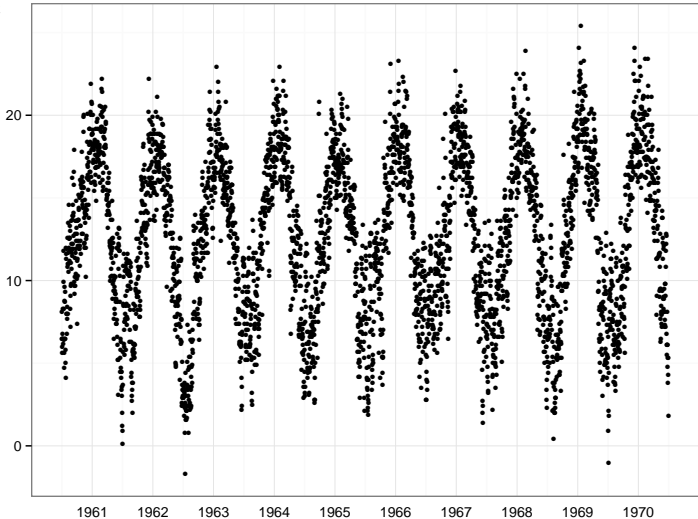
## Where does this data come from?

- ▶ The raw data have numerous **missing values** from different stations at differing points in time
- ▶ Instead we use **re-analysis** data created by Seamus Walsh at Met Éireann. These re-analysis data are available at every grid point and every time point
- ▶ They are provided on a grid of 1km by 1km, totalling 72,000 grid points with over 50 years of daily data
- ▶ Over **1.3 billion** data points altogether!

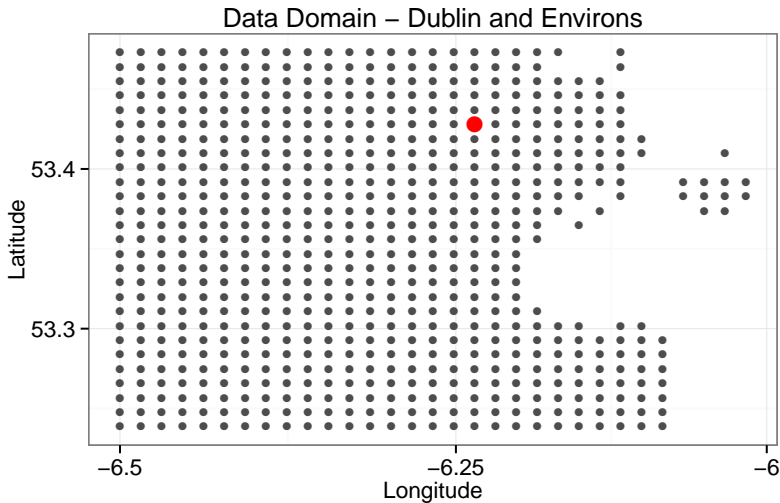
# Our data - temporal aspect

Daily Temperature at Dublin Airport

Temp in °C



## Our data - spatial aspect

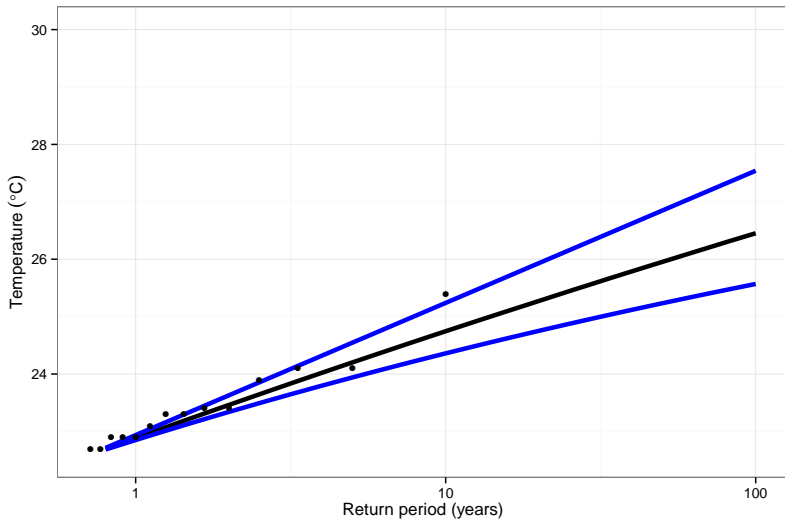


## Estimating extremes at just a single site

- ▶ Suppose we took just the daily maximum temperature at one particular site, Dublin airport
- ▶ We can fit the GEV distribution to this which will estimate the shape of the distribution curve
- ▶ From the output we can create a **return level plot** showing how likely different extreme temperatures are at different **return periods**
- ▶ The return level plot is just a function of the parameters

# The output: return level plots

Return level plot of daily maximum temperature  
at Dublin Airport 1961–1970



## A slight alternative; the Generalised Pareto distribution

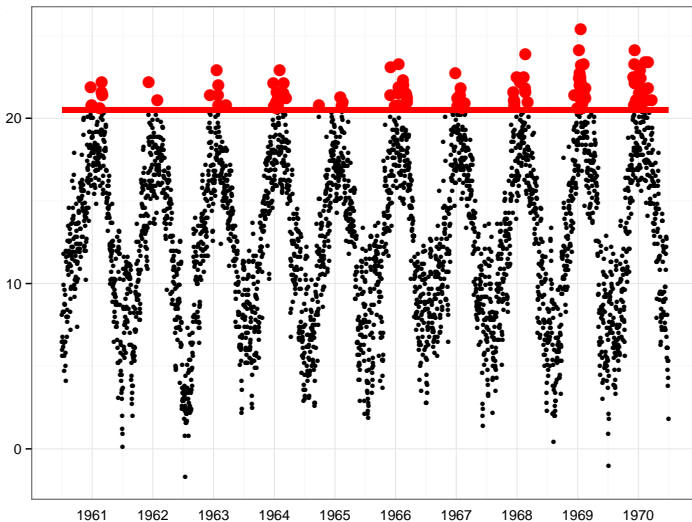
- ▶ The GEV has a slight disadvantage in that it only uses **maxima**
- ▶ An alternative, the **Generalised Pareto Distribution** (GPD) uses all the data above a threshold
- ▶ This means we get a better estimate of extremal behaviour, but we have to set a **threshold**, and can occasionally end up with more data than we can deal with
- ▶ The model fitting and output is exactly the same as the GEV



# The GPD approach

Daily Temperature at Dublin Airport

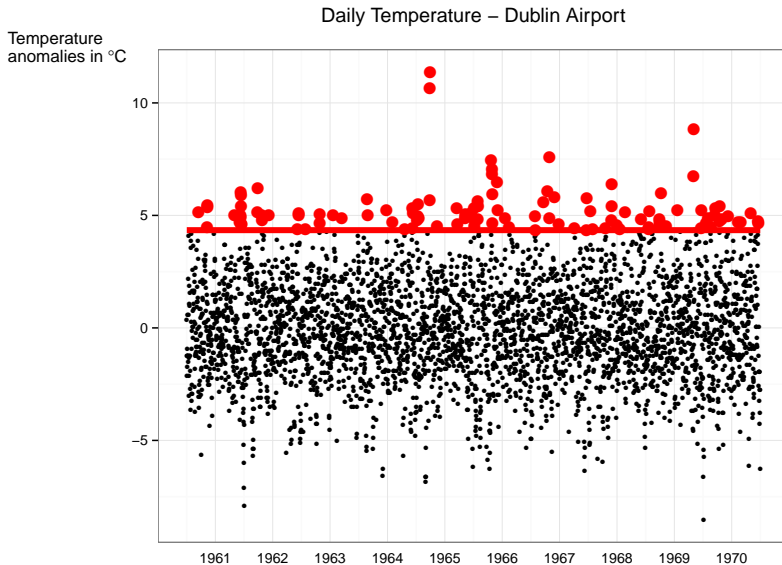
Temp in °C



## Choosing a threshold

- ▶ A tricky part of using the GPD is choosing a threshold above which you keep the data and use it to estimate extremes
- ▶ If you set the threshold too low, the assumptions of the GPD aren't valid. If you set it too high you get a really poor noisy fit
- ▶ There isn't enough information in the data usually to estimate the threshold as another parameter
- ▶ Instead it is common to choose a **high percentile** (e.g. 99%)
- ▶ With data that vary spatially or over time it might be necessary to **vary the threshold**

# Variable thresholds via anomalies

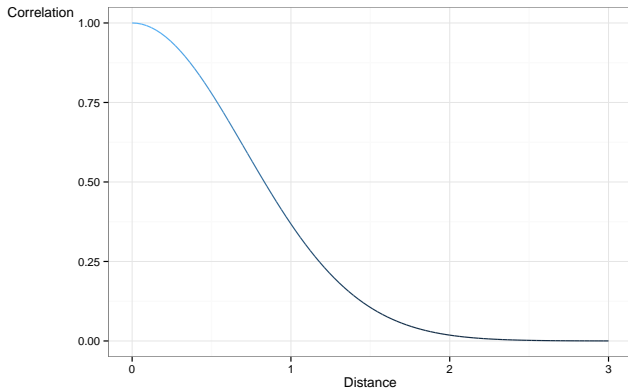


# Spatial modelling

- ▶ We have spatially indexed data so we want to take account of the fact that extremes at neighbouring sites are likely to be similar
- ▶ Spatial statistical modelling is a very mature field, however many people still run extreme value models **separately** for each site
- ▶ We use **Gaussian Processes** to force neighbouring sites to have similar extreme values

# A short introduction to Gaussian processes

- ▶ Gaussian processes work by fitting a multivariate normal distribution to the data at hand, with a restricted correlation matrix that requires high correlation of nearby sites



# Combining spatial and EVT models

We use a **hierarchical model**:

$$y_i(s) \sim GPD(\sigma(s), \xi(s)), \quad i = 1, \dots, n$$

$$\sigma(s) \sim MVN(\mu_\sigma, \Sigma_\sigma)$$

$$\xi(s) \sim MVN(\mu_\xi, \Sigma_\xi)$$

with

$$\mu_\sigma = \alpha_\sigma, \quad \mu_\xi = \alpha_\xi,$$

and

$$\Sigma_\sigma^{i,j} = \tau_\sigma^2 \rho_\sigma^{ij} + \omega_\sigma^2, \quad \Sigma_\xi^{i,j} = \tau_\xi^2 \rho_\xi^{ij} + \omega_\xi^2$$

# The advantages of a model-based Bayesian approach

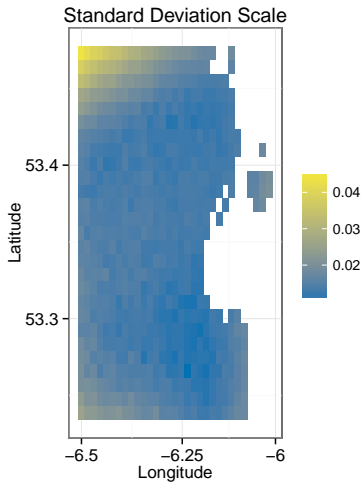
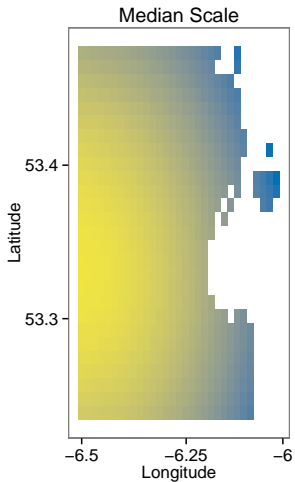
- ▶ Put everything into **one model**
- ▶ Easy to produce predictions with **uncertainty**
- ▶ Easy to combine lots of different data sets
- ▶ Possible to include informative **external information**

# Computational issues

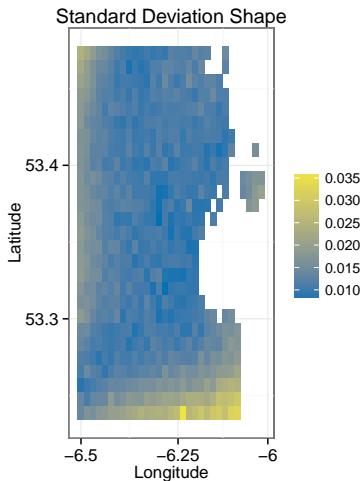
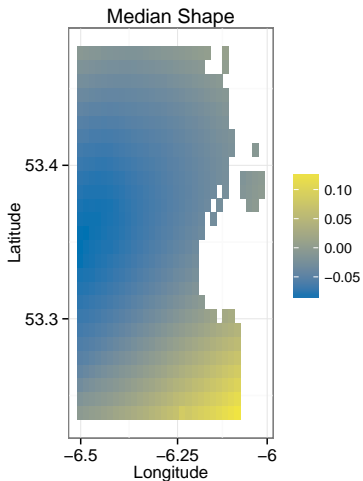
- ▶ We fit the model using **Markov chain Monte Carlo**
- ▶ This is a trial-and-error algorithm which produces probability distributions of the unknown parameters (e.g.  $\sigma(s)$  and  $\xi(s)$ ) given the data
- ▶ This probability distribution is **joint** in the sense that we have all the uncertainties and correlations between all the parameters together
- ▶ A disadvantage is that it's **slow to run** for big data sets, especially with nested Gaussian Processes because you have to solve a matrix at each step
- ▶ From the output we can use some Gaussian Processes tricks to produce predictions with uncertainty for any set of grid points we like



# Output - Dublin region for the 1960s

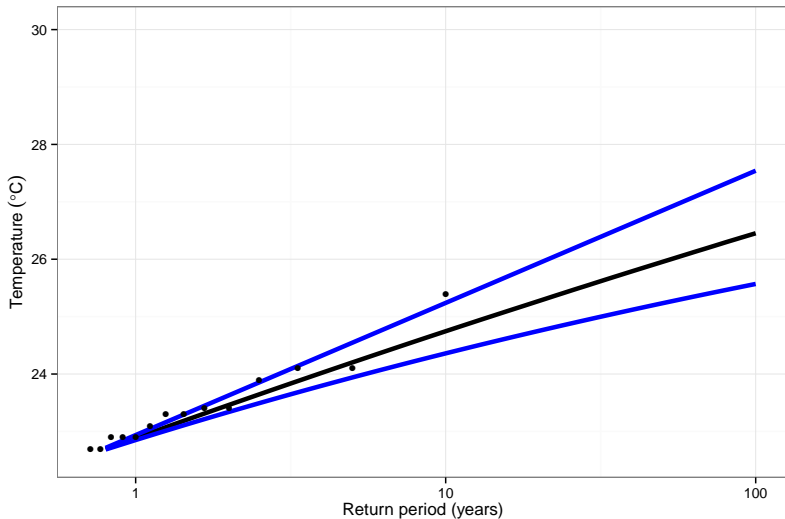


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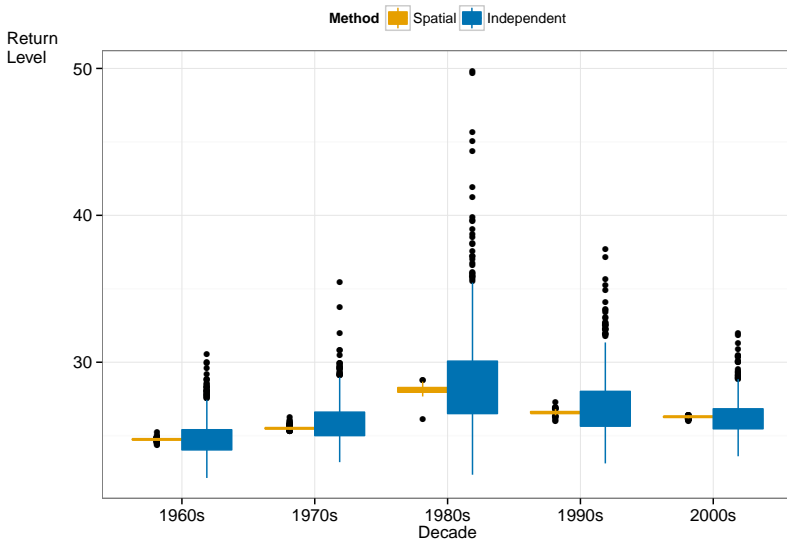
# Output - return levels for a site

Return level plot of daily maximum temperature  
at Dublin Airport 1961–1970

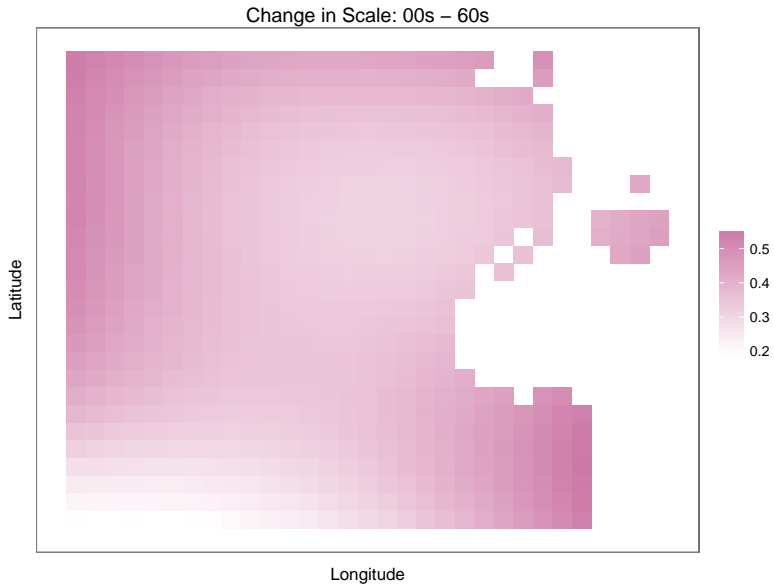


# Output - return levels over the decades

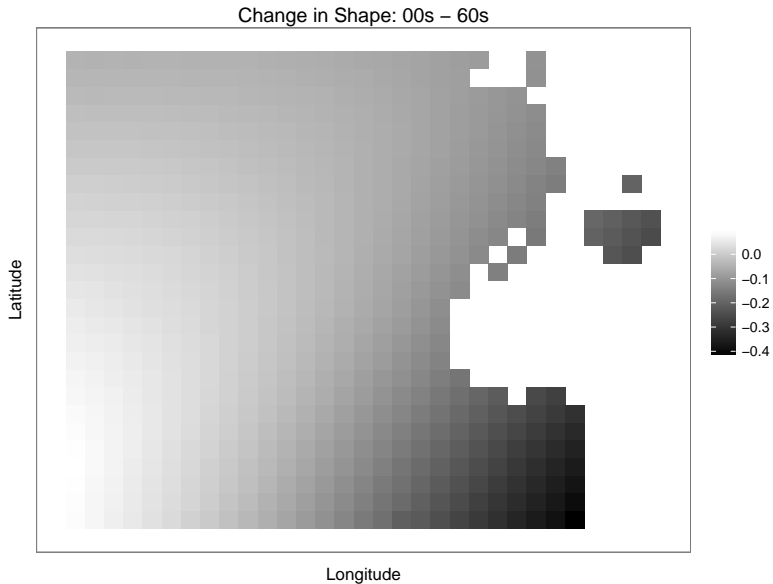
10-year return levels: Dublin Airport



# Output - changes in scale and shape



# Output - changes in scale and shape



# Spatio-temporal models

Conceptually simple to extend to **spatio-temporal** models:

$$y_i(s, t) \sim GPD(\sigma(s, t), \xi(s, t)), i = 1, \dots, n$$

$$\sigma(s, t) \sim MVN(\mu_\sigma, \Sigma_\sigma)$$

$$\xi(s, t) \sim MVN(\mu_\xi, \Sigma_\xi)$$

We then have the option of including the time series component in the mean or the covariance, or both

# Challenges:

## Statistical:

- ▶ Richer ways of **specifying spatio-temporal models**
- ▶ Richer (but more complicated) **likelihoods for extremes**
- ▶ **Faster** computational methods (SPDE-INLA, predictive processes)

## Climatological:

- ▶ Incorporating climate model data, **grid matching issues**
- ▶ **Projecting** into the future



## References and funding details

- ▶ The best book on statistical extreme value theory: Coles, S. An Introduction to the Statistical Modeling of Extreme Values, 2001, Springer
- ▶ A good paper on Hierarchical Bayesian modelling of extremes: Cooley, D., et al, Bayesian Spatial Modeling of Extreme Precipitation Return Levels, Journal of the American Statistical Association, 2005
- ▶ For the really advanced material, see, e.g. R. Huser and A. C. Davison. Space-time modelling of extreme events, in Journal of the royal statistical society: series B, 2014 and references therein

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Presentation available at: [andrewcparnell.github.io](https://andrewcparnell.github.io)